SECOND ORDER SENSITIVITY ANALYSIS AND FUNDAMENTAL FREQUENCY BASED OPTIMISATION TO PERFORM TOPOLOGY OPTIMISATION OF CONTINUUM STRUCTURES USING EVOLUTIONARY ALGORITHM

By

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ABSTRACT

Frequency based topology optimisation of continuum structures is a topic of keen interest. The main focus of this study is to propose a new method to optimise the frequency of continuum structures and perform topology optimization. A new second order approach for principal stress based sensitivity analysis using Taylor series is proposed in this study. The design objective is achieved using the Solid Isotropic Material with Penalization and Evolutionary algorithm which is used to assign the optimised relative density. The coding is done using C++ and the optimal distribution is analysed using Matlab for fundamental eigen frequency and mode shapes. The variation of normalised fundamental frequency with each iteration is studied. A few standard problems from the literature are solved and the results are compared and presented. The results show that the proposed principal stress based sensitivity analysis is quite efficient and effective compared to other methods.

Keywords: Principal stress, Sensitivity, Eigen, Frequency, Topology, Structural Optimisation, Continuum, Structures.

INTRODUCTION

Optimization of fundamental frequency is an emerging topic of interest for structural engineers. The word fundamental natural frequency means the first natural frequency for the given distribution of material in the design domain. Optimization of frequency increases the stiffness of the structure and reduces the mass of the structure. A stiff structure is the one which has least displacements when certain boundary conditions are applied (Angulo et al., 1994). The displacements are measured to find the strain energy of a structure, where the strain energy is inversely proportional to the stiffness. The method is based on an iterative process of optimization that includes structural analysis by the Finite Element Method (FEM) (Krishnamoorthy, 1994). Sensitivity analysis, and optimization techniques, the distribution of material must be effectively done to maximize the fundamental frequency. The structures with high fundamental frequency tend to be reasonably stiff for all conceivable loads and hence the optimization of fundamental frequency results in designs which are good for static loads also (Yang et al., 1999).

This paper primarily deals with the sensitivity analysis and optimization of fundamental frequency to perform topology optimization of a plate element carrying in plane loading with the given boundary conditions. Several optimization methods such as the Homogenization method, the Solid Isotropic Material with Penalization (SIMP) method, the Evolutionary Structural Optimization (ESO) method (Lee, 2012), and its later version Bi-directional ESO (BESO), the level set technique have been developed in this context. Frequency optimization is of great importance in many engineering fields e.g. aeronautical and automotive industries (Kingman et al., 2014). When compared with a large number of papers for stiffness optimization, fewer papers have been published with frequency optimization problems. When optimizing the topology of a structure it is natural to demand the solution to consist of clearly separated material and void, preferably with a material distribution possible to manufacture. This implies the use of a discrete variable for each element state, either material or void (Rozvany, 2009). The final result of a topology optimization reflects the optimal material distribution in the design space.

1. Objectives of the Study

- The main objective is to optimize the fundamental frequency and perform sensitivity analysis of the plate structure in plane stress loading condition. A combination of Finite element method and Evolutionary algorithm is to be used for optimizing the natural frequency of the structure.
- To reduce the volume of the structure.

1.1 Scope of the Study

- The study does not include buckling analysis.
- Linear static analysis is performed.
- The material obeys Hooke's law.

1.2 Structural Optimization

Structural optimization is today a very wide concept. A Structural Optimization problem achieves the best performance of the structure. A solution to a structural optimization problem is to configure the design variables in the best possible value of the performance function. The constraints include volume and weight of the structure, the structural performance such as stiffness, natural frequencies, buckling loads, maximum displacements, stresses, strain energy etc. (Van den Boom, 2014).

1.3 Topology Optimisation

A Mathematical method that optimizes the material within a given set of loads, boundary conditions and other constraints in order to maximize the system's performance and providing optimal structures (Lewinski et al., 2013). Topology is a word which means place or a position. The simple idea of topology is the removal of less efficient materials from the structure. The Topology Optimisation extends its application in structural domain for various number of holes, location, shape and their connectivity (Karadelis, 2010). The methodology of optimisation is to solve for minimum weight with stress constraints iteratively. One of the weight optimisation includes the sensitivity analysis wherein a sequence of steps are required to perform the optimisation. The first step is to perform the structural analysis using Finite Element Method and calculate the relative density (Kütük, & Göv, 2013). In the second step, the objective function is computed based on the weight of the structure. The third step requires the computation of stress constraints to verify the active constraints.

1.4 Evolutionary Structural Optimization

This approach is a combination of heuristic methods and gradient based approaches. For maximizing the stiffness, the stress is replaced by elemental strain energy criteria. In ESO approach the low sensitivity elements are not removed completely but are assigned with weak material property. The similar sensitivity scheme is employed for ESO as SIMP to avoid any numerical instabilities in the optimization problems. The ESO algorithms achieve a high quality solutions with an easy understanding and good computational efficiency.

2. Literature Review

Sarkisian et al. (2009) in their paper conveyed that the Structural Optimisation has increasing interest in building industry especially in the design of high rise buildings by distributing the members in such a way that the design efficiency can be optimized. Results show that the shape of tower which is deformed is similar to that of a cantilever beam. Two phases of optimization is performed, where in phase 1 genetic algorithm is used to identify the spacing, cable diameter, and pitch, where pitch is uniform over height of the tower. In phase 2 the pitch varies over the height of the tower and optimization is performed. Malekinejad et al. (2016) in their paper on free vibration analysis, proposed an equivalent approach. For dynamic analysis of framed tube, shear core and outrigger belt truss in high rise buildings is treated as a continuous discrete system wherein beams and columns are replaced by membranes. Lee and Bae (2010) from Gyeongsang National University from their paper described a topology optimization technique which can maximize the fundamental frequency of the structures. Paris et al. (2010) said that the basic issue in optimisation of structures is sensitivity analysis. This paper includes an efficient procedure and a complete analytic, to perform Topology optimization of continuum structures and the whole derivation process to obtain stress constraints from sensitivity analysis. The topology optimization of solid structures includes the determination of the location and shape of holes, and the connectivity of the domain. His thesis mainly constitutes with topology optimization of flat shell elements considering the four nodes, and hence the assumed strain for the shear flexible plate element and a membrane element are with drilling degrees of freedom. Zhang et al. (2015) investigated an optical design of photonic band gap structure as the model problem. The optimization problem in a domain where the material is in homogeneity so that the localization is done for an Eigen mode which in particular is governed by the scalar Helmholtz equation. Takezawa and Kitamura (2013) in his paper formulated the main objective function as minimizing the first and second eigen frequencies mode shape and the targeted mode shape by using the least squared error. Using COSMOL Multiphysics, the state and adjoint equations are solved. Tsai and Cheng (2013) in their paper proposed a technique to determine the distribution of a material of a structure. The SIMP method is used to design objective for topology optimization of continuum structures. Generally, in the optimization process, to maximize the natural frequency and prevent mode switching, the weighted constraints with bound formulation are proposed. Alavi et al. (2017) said that the observed dynamic responses in modern tall and slender structures, are considered as main design requirements instead of strength. The fundamental Eigen frequency of higher order is maximized to avoid this problem. To a real life structure, accuracy and practicality of this method is applied. The practical constraint of lower bound on stiffness is modified and added to the problem of optimization. Challis (2010) in his paper presented a new method for topology optimization that is a compact MATLAB implementation of the level set method. The code is derived such that the compliance of a statically loaded structure is minimized. The main program in short includes the initialization procedure, an iteration loop for performing the optimization and last step includes the convergence check. The function update step implements an update in the design using the shape and topological sensitivity information. Siu et al. (2003) said that the general problem for the Structural engineers is always to deal with the optimization of construction cost. The members in the design are grouped under a single category in our computer modelling and analysis. The local level constraints are used for the strength requirements of a member in moderate and large scale structures. Due to wind pressure, the structural system uses the concrete for resisting the lateral deflection. Augusto et al. (2012) in his paper constitutes two new approaches for the multi objective design optimization problems. Here the performance of the functions are highly susceptible to limited number of variations in the design variables and/or design environment parameters. Allaire et al. (2004) in the journal paper presented a new numerical method in the context of structural optimization for front propagation which is based on the combination of level set methods and classical shape derivative. On a fixed Eulerian mesh, the shape is captured and hence moderate is the cost of this numerical algorithm. Yang et al. (2015) in his paper proposed an enhanced as well as a best PSO and geometrical consistency check while tightly connecting to the ground structure approach. His contribution on MLPSO and its combination with the quadratic penalty function proved effective.

3. Methodology

The evolutionary swarm intelligence algorithms have been used to find the best optimal solutions. The initial population size is taken and the connectivity analysis is performed where in the elements are checked for edge to edge connectivity in case for 2D, leading to one of the seed elements. By edge to edge connectivity, it means that each element has a continuous edge in common with the other elements. The seed element refers to the element which always carries the material during the entire process of optimization, usually the support elements or the load carrying elements can be considered as seed elements and the presence of material in these elements must be ensured at all the times. Connectivity analysis is also performed on initial population size so as to generate better individuals. The elements which are having corner connectivity are not considered as corners as they cannot transfer any moment.

The initial connectivity analysis followed by finite element analysis will help to generate the structures that are stable and have better values. This optimizer algorithm was proposed by Xin She Yang in 2009 by the name of Levy firefly algorithm (Yang, 2010). The objective function is formulated for a set of equations and analyzed.

$$r_{ij}=\sqrt{\sum_{k=1}^{d}(x_{i,k}-x_{j,k})^2}$$

where, $x_{i,k}$ is the k^{th} component of the spatial co-ordinate x_i of i^{th} firefly

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2}(x_j - x_i) + \alpha\, sign\left[rand - \frac{1}{2}\right](Levy + drho) \quad (1)$$

Levy Paul Pierre distribution, $Levy=\frac{1}{\sqrt{2\Pi}}\frac{e^{\frac{-u}{2}}}{u^{3/2}}$

The steps of firefly is drawn from a levy distribution random walk with a power law step length distribution with a heavy tail. drho is the sensitivity factor. The plate structure is initially discretized into a number of finite elements using quadrilateral elements. Each element comprises of four nodes and each node has two degrees of freedom. The plate is defined with material properties, kinematic boundary conditions, loading and support conditions with certain constraints. The finite element analysis is performed and there by the Stiffness matrix is generated. As the force vector is known the displacement matrix is determined for the continuum. The stresses and strains are determined, and the objective function for the initial value is calculated. The Eigen values are the eigen frequencies which are obtained by LDLT transformation (Sundar & Bhagavan, 2000) and by reducing to a standard format. The Arnold algorithm is used for a tridiagonal format. The objective function is to perform the frequency optimization. The firefly algorithm is used as an optimizer to perform stress based sensitivity analysis. The mass of the material eventually gets reduced as the frequency is maximized. The relative density of the each element is calculated, and the elements are differentiated based on the black and white colours where the black represents the full material which carry the load, and the white colour '0' represents no load. The distribution of the material having better value is generated. A graph with fundamental frequency on Y axis versus the iteration number on X axis is plotted. The best iterated value is presented.

3.1 Problem Statement

The problem statement is defined here in this section as follows.

Maximize fundamental frequency λ

subject to,

$$\sigma_{Prin,e} - \sigma_{Allow} \le 0$$

$$u_{Nodal} - u_{Allow} \le 0$$

$$0 \le \rho_{min} \le \rho_e \le 1$$

$$(K-\lambda_j M)\,\Phi_j = 0, \text{ where } j=1$$

$$\left(\sum_{i=1}^{n}\rho_i \upsilon_i\right) * \bar{\rho} > 0$$

The objective is to optimize using the fundamental frequency and find the minimum weight of the structure subject to the constraints on the amount of material used and the elemental centroidal stresses and nodal displace-ments in the structure.

The corresponding global stiffness matrix (K) and mass matrix (M) are given by,

$$K=\sum_{e=1}^{Ne}\rho_e^p k_e$$

$$K=\sum_{e=1}^{Ne}\rho_e^q m_e$$

4. Theoretical Background

Sensitivity analysis is presented accordingly as explained.

4.1 Sensitivity Analysis

We propose Sensitivity analysis using the principal stresses here.

The governing equation is given by,

$$K(\rho)\alpha(\rho)=f(\rho)$$

where K is the stiffness matrix, f is the force vector which are functions of the relative density.

The vector of displacements α can be obtained as,

$$K\frac{d\alpha}{d\rho}=\frac{df}{d\rho}-\frac{dK}{d\rho}\alpha \qquad (2)$$

4.2 Proposed Sensitivity Analysis using Principal Stresses

At the centroid of an element, the stress is given by,

$$[\sigma_{CEN}] = \begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \rho^p\, D\, B^{CENTROID}\alpha \qquad (3)$$

where 'D' is the elasticity matrix, B is the strain displacement matrix, α is the displacement vector, 'ρ' is the relative density of each element and 'p' is the penalization factor. The normal and tangential stresses on any plane inclined at angle θ with the plane carrying normal stress σ_x and accompanied by a shear stress τ_{xy} is given in Figure 1.

Consider the force equilibrium on the elementary portion for the plane AC having unit thickness and inclined at an angle θ with the vertical AB, we have,

Forces perpendicular to the plane AC,

$\sigma_x AB\cos\theta + \tau_{xy} AB\sin\theta + \sigma_y BC\sin\theta + \tau_{xy} BC\cos\theta$

Forces parallel to the plane AC,

$\sigma_x AB\cos(90-\theta) - \tau_{xy} AB\cos\theta - \sigma_y BC\cos\theta + \tau_{xy} BC\cos(90-\theta)$

The triangle ABC is a right angled triangle,

$AB=AC\cos\theta$, $BC=AC\sin\theta$

The normal stress on the inclined plane AC is given by,

$$\sigma_{NORMAL} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

The tangential stress on the inclined plane AC is given by,

$$\sigma_{TANGENTIAL} = \frac{\sigma_x-\sigma_y}{2}\sin(2\theta) - \tau_{xy}\cos(2\theta)$$

To determine the principal planes, the tangential stress is set equal to zero.

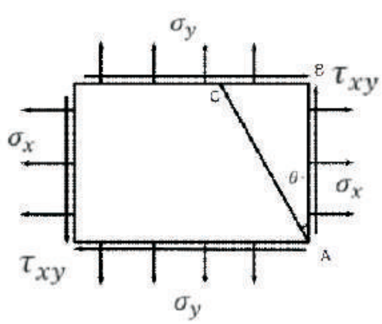

Figure 1. Stresses on a Plane

$$\tan(2\theta_1) = \frac{2\tau_{xy}}{\sigma_x-\sigma_y} \quad and\ \theta_2 = \theta_1 + 90^0$$

Determine the Principal stresses σ_1 and σ_2.

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos(2\theta_{1,2}) + \tau_{xy}\sin(2\theta_{1,2})$$

$$\sigma_{Prin\ 1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (4)$$

4.3 First Order Sensitivity Analysis

$$[\sigma_{CEN}] = \begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \rho^p\, D\, B^{CENTROID}\alpha$$

$$\frac{d\sigma_{CEN}}{d\rho} = \frac{d\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}}{d\rho} = D\,B^{CENTROID}\left\{\rho^p\frac{\partial\alpha}{\partial\rho} + \alpha\, p\, \rho^{p-1}\frac{\partial\rho}{\partial\rho}\right\}$$

$$where\ \frac{\partial\alpha}{\partial\rho} = K^{-1}\left\{\frac{dF}{d\rho} - \frac{dK}{d\rho}\alpha\right\}$$

$$\frac{dK}{d\rho} = p\ \rho^{p-1}\left(\sum_1^{GP} B^T DB\right)$$

$$\frac{dK}{d\rho}\frac{d\alpha}{d\rho} + K\frac{\partial^2\alpha}{\partial\rho^2} = \frac{\partial^2 F}{\partial\rho^2} - \frac{dK}{d\rho}\frac{d\alpha}{d\rho} - \frac{\partial^2 K}{\partial\rho^2}\alpha$$

$$\frac{\partial^2\alpha}{\partial\rho^2} = K^{-1}\left[\frac{\partial^2 F}{\partial\rho^2} - 2\frac{dK}{d\rho}\frac{d\alpha}{d\rho} - \frac{\partial^2 K}{\partial\rho^2}\alpha\right]$$

$$\frac{\partial^2 K}{\partial\rho^2} = (p)\,(p-1)\ \rho^{p-2}\left(\sum_1^{GP} B^T DB\right)$$

$$\frac{\partial^2\sigma_{CEN}}{\partial\rho^2} = \frac{\partial^2\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}}{\partial\rho^2} = DB^{CEN}\left[2\,p\,\rho^{p-1}\frac{\partial\alpha}{\partial\rho} + \rho^p\frac{\partial^2\alpha}{\partial\rho^2} + (p)(p-1)\rho^{p-2}\alpha\right] \qquad (5)$$

$$\frac{d\sigma_{prin}}{d\rho} = \frac{d\sigma_x}{2\,d\rho} + \frac{d\sigma_y}{2\,d\rho} \pm \frac{1}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}\left[2\left(\frac{\sigma_x-\sigma_y}{2}\right)\frac{1}{2}\left(\frac{d\sigma_x}{d\rho}-\frac{d\sigma_y}{d\rho}\right) + 2\tau_{xy}\frac{d\tau_{xy}}{d\rho}\right]$$

Let,

$$\frac{u}{v} = \frac{1}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}\left[2\left(\frac{\sigma_x-\sigma_y}{2}\right)\frac{1}{2}\left(\frac{d\sigma_x}{d\rho}-\frac{d\sigma_y}{d\rho}\right) + 2\tau_{xy}\frac{d\tau_{xy}}{d\rho}\right]$$

$$d\left(\frac{u}{v}\right) = \frac{vdu-udv}{v^2}$$

$$du = \left(\frac{\sigma_x-\sigma_y}{2}\right)\left(\frac{d^2\sigma_x}{d\rho^2}-\frac{d^2\sigma_y}{d\rho^2}\right) + \frac{\left(\frac{d\sigma_x}{d\rho}-\frac{d\sigma_y}{d\rho}\right)\left(\frac{d\sigma_x}{d\rho}-\frac{d\sigma_y}{d\rho}\right)}{2} + 2\tau_{xy}\frac{d^2\tau_{xy}}{d\rho^2} + 2\frac{d\tau_{xy}}{d\rho}\frac{d\tau_{xy}}{d\rho}$$

$$dv = \frac{1}{2\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}\left\{2(\sigma_x-\sigma_y)\left(\frac{d\sigma_x}{d\rho}-\frac{d\sigma_y}{d\rho}\right) + 8\tau_{xy}\frac{d\tau_{xy}}{d\rho}\right\}$$

4.4 Second Order Sensitivity Analysis

$$\frac{d^2\sigma_{Prin}}{d\rho^2} = \frac{d^2\sigma_x}{2\,d\rho^2} + \frac{d^2\sigma_y}{2\,d\rho^2} + \frac{vdu-udv}{v^2} \qquad (6)$$

4.5 Taylor's Series

We use the following approach of second order approximation using Taylor's series,

Minimize $f(x_k)$.

Using Taylor series, we have,

$$f(x_k+\Delta x_k) = b + c^T\Delta x + \frac{1}{2}\Delta x^T H\,\Delta x$$
$$= f(x_k) + \Delta x\,\nabla f(x_k)^T + \frac{1}{2}\Delta x^T H\,\Delta x$$
$$= f(x_k) + \Delta x\, f'(x_k) + \frac{1}{2}\Delta x^T H\,\Delta x$$

For maximum or minimum, first derivative is equal to zero.

$$\nabla f(x_k+\Delta x_k) = f'(x_k) + H\Delta x = \nabla f(x_k) + H\Delta x = 0$$
$$\Delta x = -H^{-1} f'(x_k) = -H^{-1}\,\nabla f(x_k)$$

The change in the relative density of the element is,

$$d\rho = -\left(\frac{d^2\sigma_{Prin}}{d\rho^2}\right)^{-1}\left(\frac{d\sigma_{Prin}}{d\rho}\right)$$

5. Analysis

We proposed a new sensitivity analysis, a set of individuals search the de-sign domain and determine the optimum values of the relative densities for every element. The advantage of using this approach is to determine the distribution of material which not only satisfy the boundary conditions namely natural boundary conditions, and kinematic boundary conditions but also perform the optimization based on fundamental frequency of the material distribution as shown in the Figure 2. The final distribution of material is frequency optimized and also satisfy the boundary conditions at the convergence. As a result, the weight of the structure is also reduced. In this section, a few examples are solved using the proposed formulation based on the sensitivity analysis using the principal stress at the centroid of each element. The final distribution of the material obtained from C++ program is smoothened in Auto CAD® and analysed using MATLAB®.

5.1 Standard Benchmark Problems

5.1.1 Problem 1

A cantilever plate is fixed at the left-top and left-bottom. As shown in the Figure 3 the plate is 0.16 m by 0.10 m in size. The domain is discretized using 16 x 10 = 160 first order four node quadrilateral elements. The total number of nodes is 187. The cantilever plate carries an in plane loading of P as 3000 N at the bottom node point on the right side. The Young's modulus of the material is equal to 200 GPa and the Poisson's ratio is equal to 0.33. The thickness of the plate is taken as 1 unit. The allowable stress for the material is taken as 200 MPa (Luh et al., 2011).

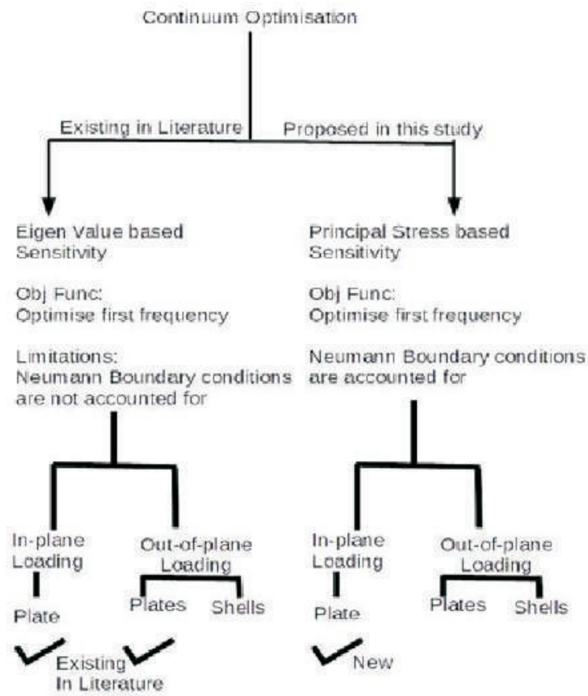

Figure 2. The Proposed Sensitivity Analysis

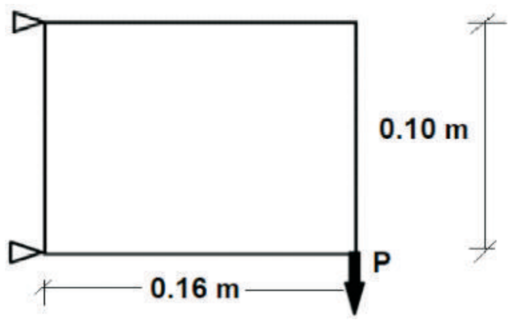

Figure 3. The Cantilever Plate is Fixed at the Left-top and Left-bottom

The static displacement is as shown in the Figure 4. The maximum Y-displacement is found to be 0.74 mm. Figure 6 shows the variation of the normalized fundamental frequency with each iteration. The maximum value of first frequency is 533.249% higher than the initial frequency.

The optimization process took 87 functional evaluation computations using the proposed method when compared to 36000 functional evaluations by Guan using particle swarm optimization as shown in Table 1. The volume of material in the optimal distribution by Guan is 58.125% of initial volume when compared to the volume of optimal distribution 32.5% of the initial volume using Eigen FFA study. Figure 5 shows the fundamental mode shape and the square of the fundamental frequency is 67100 rad^2/s^2. Lin and Hsu (2008) has optimized a similar problem in Foot Pound Second (FPS) system.

Figure 4. The Y-displacements

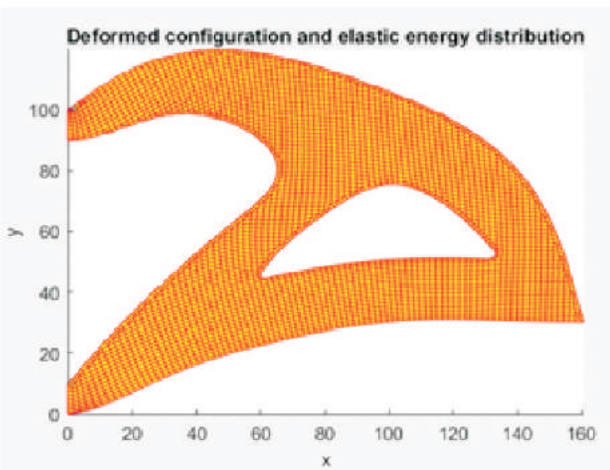

Figure 5. The First Mode Shape

	Luh et al. (2011) (Case 2)	This Study using Eigen FFA
Final Volume (%)	58.125	32.5
Number of FE computations	36000	87

Table 1. Comparison of the Volume After Optimisation.

5.1.2 Problem 2

A Cantilever plate carrying a point load at the 2/5th distance from the right bottom corner. A cantilever plate is fixed at the left-top and left-bottom as shown in the Figure 7. The plate is 0.16 m by 0.10 m in size. The domain is discretized using 16 x 10 = 160 first order four node quadrilateral elements. The total number of nodes is 187. The cantilever plate carries an in plane loading of P=3000 N at a distance of 2/5th from the bottom node point on the right side. The Young's modulus of the material is equal to 200 GPa and the Poisson's ratio is equal to 0.33. The thickness of the plate is taken as 1 unit. The allowable stress for the material is taken as 200 MPa (Luh et al., 2011).

Figure 8 shows the static Y-displacement for the final distribution. The maximum Y-displacement is found to be

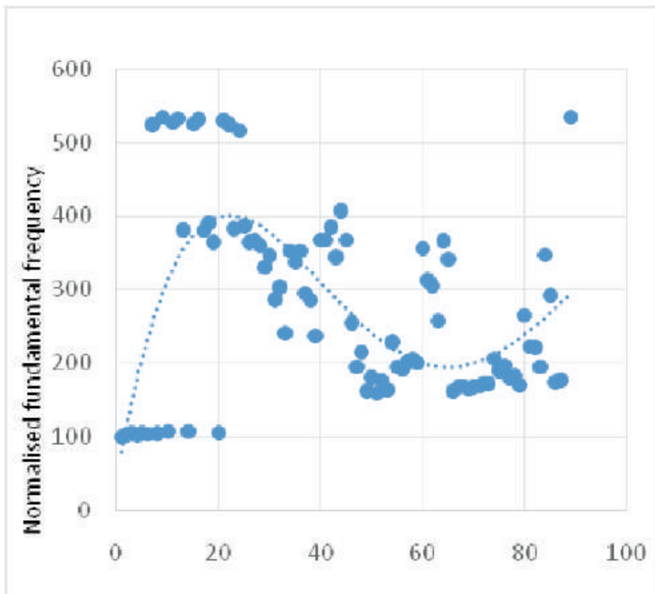

Figure 6. Graph Showing the Variation of Normalized Fundamental Frequency on Y-axis with Iteration on X-axis for a Cantilever Carrying a Point Load at the Corner

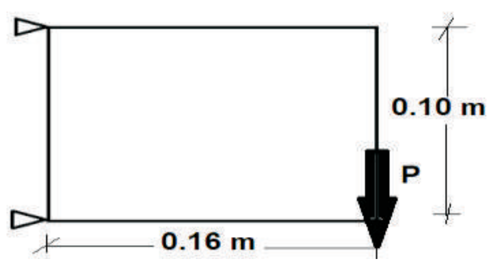

Figure 7. The Cantilever Plate Carrying a Point Load at the 2/5th Distance from the Corner

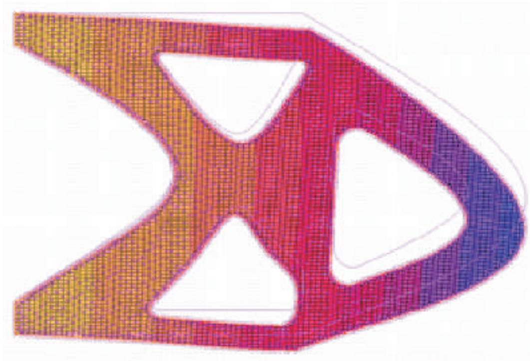

Figure 8. The Displacement in Y-Direction

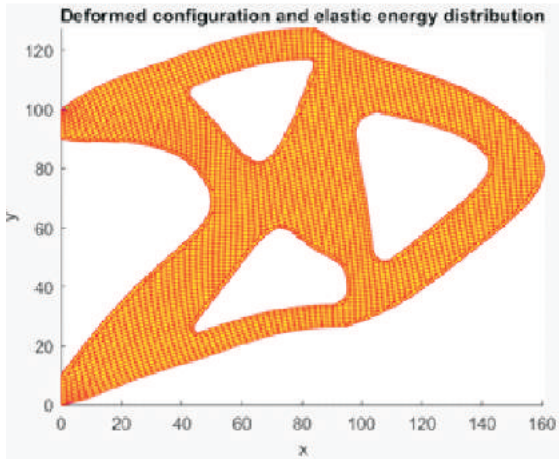

Figure 9. The First Mode Shape

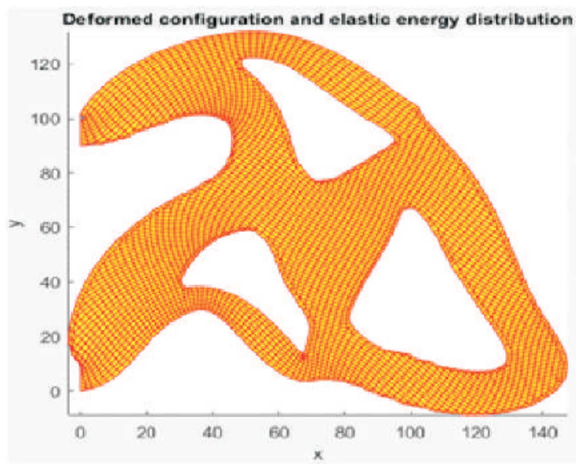

Figure 10. The Second Mode Shape

0.69 mm in the vertically downward direction. Figure 11 shows the variation of the normalized fundamental frequency with each iteration. The maximum value of first frequency is 501.59% higher than the initial frequency. The optimization process took 142 functional evaluation computations using the proposed method when compared to 36000 functional evaluations by Guan using particle swarm optimization as shown in Table 2. The volume of material in the optimal distribution is 35.625% as compared to 57.968% of the initial volume by Guan. Figure 9 and Figure 10 present the first mode and the second mode of vibration respectively. The square of the fundamental frequency and second frequency of vibration are 106900 rad^2/s^2, 528600 rad^2/s^2 respectively.

6. Future Study

The present study can be further extended to perform the frequency optimisation of structures discretized using first order six node hexagonal elements as shown in the Figure 12.

The frequency optimisation of shell structures is also an emerging area of research in the field of structural engineering and optimisation.

The square of the optimal frequencies are 95.8619 rad^2/s^2 and 121.4242 rad^2/s^2 for the first mode and second mode of vibration respectively.

7. Discussion of Results

In addition to the frequency optimisation the final volume of material carrying the loads is also reduced during the optimisation process. The positive results gave an insight to develop algorithm which is presented here.

For a cantilever plate which is fixed at the left-top and left-bottom corner and carrying the load at the right hand bottom corner the frequency is maximized by 533.249%. The square of the fundamental frequency is 67100 rad^2/s^2. The final volume of the material is 32.5% in 87 iterations compared to final volume of 58.125% by Guan using Particle Swarm Optimisation algorithm in 36000 iterations.

	Luh et al. (2011) (Case 4)	This Study using Eigen FFA
Final Volume (%)	57.96875	32.625
Number of FE computations	36000	142

Table 2. Comparison of the Volume After Optimisation

For a cantilever plate carrying a point load at the 2/5th distance from the right bottom corner, the frequency is maximized by 501.59% higher than the initial frequency. The square of the fundamental frequency and second frequency of vibration are 106900 rad^2/s^2, 528600 rad^2/s^2 respectively. The final volume of the material is 35.625% in

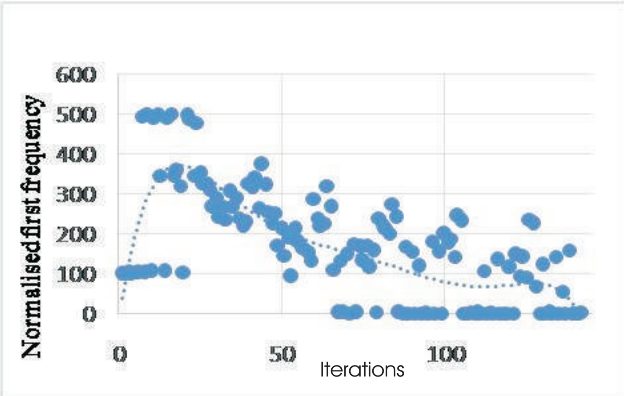

Figure 11. Graph Showing the Variation of Normalized First Frequency on Y-axis with Iteration on X-axis for a Cantilever Carrying Load at 2/5th Distance from the Corner

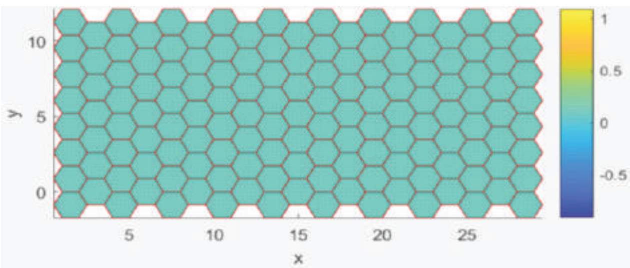

Figure 12. Showing the Domain Mesh using Six Node First Order Hexagon Element

142 iterations compared to final volume of 57.968% obtained by Guan using Particle Swarm Optimisation algorithm in 36000 iterations.

Conclusion

Topology optimization has been the structural engineer's topic of research in the recent past over the last sixty years. The objective of this paper, is to propose stress based second order sensitivity analysis and use Firefly algorithm to optimize the frequency of continuum structures. The Metaheuristic Firefly algorithm can optimize not only faster but also computationally less expensive. The higher order derivatives are important in determining changes in parameter variations using Taylor series expansion. In particular, second order derivatives are also used in optimization techniques and stability analysis. The ESO method is very simple to program via. the FEA packages and requires a relatively small amount of time. The elements that are not carrying any load are removed and consequently constitute relatively fully stressed designs. The topology optimization technique for optimizing the fundamental frequency of the structure is used to produce optimum topologies of the continuum structures. A few sample problems have been solved and we found that the output of this research when compared with those existing in the literature leads to lowest weight distribution of material.

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